Generalized Likelihood Ratio Test:

The generalized likelihood ratio test is denoted by $'\lambda'$ is defined to be

$$\lambda = \lambda_n = \frac{\theta^{\sup} \varepsilon \underset{-0}{\bar{\theta}} \; L(\theta, X)}{\theta^{\sup} \varepsilon \underset{-0}{\bar{\theta}} \; L(\theta, X)}$$

The value $'\lambda'$ are used to formula C test.

$$H_0 : \theta \varepsilon \underset{-}{\bar{\theta}_0} \qquad \text{Vs} \qquad H_A : \theta \varepsilon \underset{-}{\bar{\theta}} - \underset{-}{\bar{\theta}_0}$$

By generalized likelihood ratio test Ho is rejected if $\lambda \le \lambda_0$ if some fixed constant satisfying?

Uniformly Most Powerful Test:

The region 'C' is called uniformly most powerful critical region of size $'\alpha'$ and the corresponding test as UMP of level $'\alpha'$ for testing simple $H_0 : \theta = \mu_0$ against composite $H_1 : \theta \ne \mu_0$.

Uniformly Most Powerful Test:

A test $\hat{r}$ of $H_0 : \theta \varepsilon \underset{-}{\bar{\theta}_0}$ Vs $H_A : \theta \varepsilon \underset{-}{\bar{\theta}} - \underset{-}{\bar{\theta}_0}$ is defined to be uniform most powerful of size α if

1) $\theta^{\sup} \varepsilon \underset{-0}{\bar{\theta}} \; \pi_{\hat{r}}(\theta) = \alpha$

2) $\pi_{\hat{r}}(\theta) \ge \pi_{\hat{r}}(\theta)$ For all $\theta \varepsilon \underset{-}{\bar{\theta}} - \underset{-}{\bar{\theta}_0}$ and for any test r with size less than or equal to α.

OR

A test $\hat{r}$ is UMP of size 'α' if it has size 'α' and if among all tests its size is less than or equal to

'α'. It has the largest power of all alternative values of θ.

Minimax Test:

A test r_m of $H_0 : \theta = \theta_0$ Vs $H_1 : \theta = \theta_1$ is defined to be minimax,

If

$$Max[R_{rm}(\theta_0) - R_{rm}(\theta_1)] \le Max[R_r(\theta_0), R_r(\theta_1)]$$

Bays Test:

A test r_g of $H_0 : \theta = \theta_0$ Vs $H_1 : \theta = \theta_1$ is defined to be Bays test with respect to prior distribution

Iff

$$(1-g)R_{r_g}(\theta_0) + gR_{r_g}(\theta_1) \le (1-g)R_r(\theta_0) + gR_r(\theta_1)$$

Iff any other test 'r'.

Question 1:

Let $X_1, X_2, X_3, ..., X_n$ **be a random sample the distribution** $N(0,\theta)$ **where** $'\theta'$ **is unknown and +ve number. Show that sample** $H_0: \theta = \theta'$ **Vs** $H_1: \theta > \theta'$ **by UMPT.**

As X~ $N(0,\theta)$

$$f(x) = \frac{1}{\sqrt{\theta}\sqrt{2\pi}} e^{-\frac{1}{2\theta}X^2}$$

$H_0: \theta = 3$ Let $H_0: \theta = \theta'$

$H_1: \theta > 3$ Let $H_1: \theta > \theta'$

And X~ $N(0,\theta)$

$$f(x) = \frac{1}{\sqrt{\theta}\sqrt{2\pi}} e^{-\frac{1}{2\theta}X^2}$$

Taking likelihood function

$$L(\underline{x}) = (\frac{1}{\sqrt{\theta}})^n (\frac{1}{\sqrt{2\pi}})^n e^{-\frac{1}{2\theta}\Sigma X^2}$$

As $H_0: \theta = \theta'$

$$L(\theta') = (\frac{1}{\sqrt{\theta'}})^n (\frac{1}{\sqrt{2\pi}})^n e^{-\frac{1}{2\theta'}\Sigma X^2}$$

As $H_1: \theta > \theta'$

Let θ'' **be a value such that** $\theta'' > \theta'$ **then** $H_1: \theta = \theta''$

Then

$$L(\theta'') = (\frac{1}{\sqrt{\theta''}})^n (\frac{1}{\sqrt{2\pi}})^n e^{-\frac{1}{2\theta''}\Sigma X^2}$$

By Nyman Pearson Lemma Theorem

$$\frac{L(\theta')}{L(\theta'')} \le K_\alpha$$

$$\frac{(1/\sqrt{\theta'})^n (\frac{1}{\sqrt{2\pi}})^n e^{-\Sigma X^2/2\theta'}}{(1/\sqrt{\theta''})^n (\frac{1}{\sqrt{2\pi}})^n e^{-\Sigma X^2/2\theta''}} \le K_\alpha$$

$$(\frac{\theta''}{\theta'})^{n/2} e^{-\Sigma X^2/2\theta' + \Sigma X^2/2\theta''} \le K_\alpha$$

$$e^{-\Sigma X^2/2\theta' + \Sigma X^2/2\theta''} \le K_\alpha (\frac{\theta'}{\theta''})^{n/2}$$

Taking log on both sides.

$$-\frac{\sum X^2}{2\theta'}+\frac{\sum X^2}{2\theta''}\le \log\left[K_\alpha(\frac{\theta'}{\theta''})^{n/2}\right]$$

$$-\frac{\sum X^2}{2}\left[\frac{1}{\theta'}-\frac{1}{\theta''}\right]\le \log\left[K_\alpha(\frac{\theta'}{\theta''})^{n/2}\right]$$

$$-\frac{\sum X^2}{2\theta'\theta''}(\theta''-\theta')\le \log\left[K_\alpha(\frac{\theta'}{\theta''})^{n/2}\right]$$

$$-\sum X^2\le 2\theta'\theta''\log\left[K_\alpha(\frac{\theta'}{\theta''})^{n/2}\right]/(\theta''-\theta')$$

$$\sum X^2\ge -2\theta'\theta''\log\left[K_\alpha(\frac{\theta'}{\theta''})^{n/2}\right]/(\theta''-\theta')$$

$$\therefore\ C=-2\theta'\theta''\log\left[K_\alpha(\frac{\theta'}{\theta''})^{n/2}\right]/(\theta''-\theta')$$

$$\sum X^2\ge C$$

It is uniform most powerful critical region (UMPCR) to test $H_0:\theta=\theta'$ **Vs** $H_1:\theta=\theta''$ **of size** α **.**

Question 2:

Let $X_1, X_2, X_3,..., X_n$ **be a random sample the distribution** $N(0,\theta)$ **where** $'\theta'$ **is unknown and +ve number. Show that sample** $H_0:\theta=3$ **Vs** $H_1:\theta>3$ **by UMPT. If n=15 and** α **=0.05 . Find the value of C.**

Solution:

$$P\left[\sum_{i=1}^{n}X_i^2\ge\frac{C}{H_0}\right]=\alpha$$

$$P\left[\sum_{i=1}^{15}\frac{(X-0)^2}{3}\ge\frac{C}{3}\right]=\alpha \qquad \therefore H_0:\theta=3$$

$$P\left[\chi^2_{(15)}\ge\frac{C}{3}\right]=0.05$$

$$P\left[\chi^2_{(15)}\ge 25\right]=0.05$$

So C/3=25

C=75

Hence $\sum X_i^2\ge 75$ **is uniform Most Powerful critical region to test** $H_0:\theta=3$ **Vs** $H_1:\theta>3$

Question 3:

Let $X_1, X_2, X_3, ..., X_n$ **be a random sample the distribution** $N(\theta,1)$ **where** $'\theta'$ **is unknown show that there is no uniform most powerful test** $H_0 : \theta = \theta'$ **Vs** $H_1 : \theta \neq \theta'$ **.**

As x~ $N(\theta,1)$

$$f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}(X-\theta)^2}$$

Taking likelihood function

$$L(\underline{x}) = (\frac{1}{\sqrt{2\pi}})^n e^{-\frac{1}{2}\Sigma(X-\theta)^2}$$

As $H_0 : \theta = \theta'$

$$L(\theta') = (\frac{1}{\sqrt{2\pi}})^n e^{-\frac{1}{2}\Sigma(X-\theta')^2}$$

AS $H_1 : \theta \neq \theta'$

Let θ'' is a value such that $\theta'' \neq \theta'$ .then

$H_1 : \theta = \theta''$

$$L(\theta'') = (\frac{1}{\sqrt{2\pi}})^n e^{-\frac{1}{2}\Sigma(X-\theta'')^2}$$

By Nyman Pearson Lemma Theorem

$$\frac{L(\theta')}{L(\theta'')} \leq K_\alpha$$

$$\frac{(\frac{1}{\sqrt{2\pi}})^n e^{-\frac{1}{2}\Sigma(X-\theta')^2}}{(\frac{1}{\sqrt{2\pi}})^n e^{-\frac{1}{2}\Sigma(X-\theta')^2}} \leq K_\alpha$$

$$e^{-\frac{1}{2}\Sigma x^2 - \frac{1}{2}n\theta'^2 + n\bar{x}\theta' + \frac{1}{2}\Sigma x^2 + \frac{1}{2}n\theta''^2 - n\bar{x}\theta''} \leq K_\alpha$$

$$e^{-\frac{1}{2}n\theta'^2 + \frac{1}{2}n\theta''^2 + n\bar{x}\theta' - n\bar{x}\theta''} \leq K_\alpha$$

Taking log on both sides

$$-\frac{1}{2}n\theta'^2 + \frac{1}{2}n\theta''^2 + n\bar{x}\theta' - n\bar{x}\theta'' \leq \log K_\alpha$$

$$-\frac{1}{2}n\theta'^2+\frac{1}{2}n\theta''^2+n\bar{x}(\theta'-\theta'')\le \log K_\alpha$$

$$\bar{x}(\theta'-\theta'')\le \frac{1}{n}\left[\log K_\alpha+\frac{1}{2}n\theta'^2-\frac{1}{2}n\theta''^2\right]$$

Case 1:

If $\theta''>\theta'$

$$-\bar{x}(\theta'-\theta'')\le \frac{1}{n}\left[\log K_\alpha+\frac{1}{2}n\theta'^2-\frac{1}{2}n\theta''^2\right]$$

Multiply by '-1' on both sides

$$\bar{x}\ge -\frac{1}{n((\theta'-\theta''))}\left[\log K_\alpha+\frac{1}{2}n\theta'^2-\frac{1}{2}n\theta''^2\right]$$

$$\therefore C_1=-\frac{1}{n((\theta'-\theta''))}\left[\log K_\alpha+\frac{1}{2}n\theta'^2-\frac{1}{2}n\theta''^2\right]$$

$$\bar{x}\ge C_1$$

Is required uniform most powerful critical region (UMPCR) when $\theta''>\theta'$ **.**

Case 2:

If $\theta''<\theta'$

$$\bar{x}(\theta'-\theta'')\le \frac{1}{n}\left[\log K_\alpha+\frac{1}{2}n\theta'^2-\frac{1}{2}n\theta''^2\right]$$

$$\bar{x}\le \frac{1}{n((\theta'-\theta''))}\left[\log K_\alpha+\frac{1}{2}n\theta'^2-\frac{1}{2}n\theta''^2\right]$$

$$\therefore C_2=\frac{1}{n((\theta'-\theta''))}\left[\log K_\alpha+\frac{1}{2}n\theta'^2-\frac{1}{2}n\theta''^2\right]$$

$$\bar{x}\le C_2$$

Is required uniform most powerful critical region (UMPCR) .hence there is no UMPCR for $H_1:\theta\ne\theta''$**.**

Question 4:

Examine whether a BCR exist for testing $H_0:\theta=\theta_0$ **Vs** $H_1:\theta>\theta_0$

For $f(x)=\dfrac{1+\theta}{(x+\theta)^2}$ $\qquad 1\le x\le\infty$

$$f(x)=\frac{1+\theta}{(x+\theta)^2}$$

Taking likelihood function

$$L(\underline{x}) = \frac{(1+\theta)^n}{\prod_{i=1}^{n}(x+\theta)^2} \quad H_0 : \theta = \theta_0$$

As $H_0 : \theta = \theta_0$

$$L(H_0) = \frac{(1+\theta_0)^n}{\prod_{i=1}^{n}(x+\theta_0)^2}$$

As $H_1 : \theta > \theta_0$

Let θ_1 be a value that $\theta_1 > \theta_0$ then $H_1 : \theta = \theta_1$

$$L(H_1) = \frac{(1+\theta_1)^n}{\prod_{i=1}^{n}(x+\theta_1)^2}$$

By Nyman Pearson Lemma Theorem

$$\frac{L(\theta_0)}{L(\theta_1)} \le K_\alpha$$

$$\frac{\dfrac{(1+\theta_0)^n}{\prod_{i=1}^{n}(x+\theta_0)^2}}{\dfrac{(1+\theta_1)^n}{\prod_{i=1}^{n}(x+\theta_1)^2}} \le K_\alpha$$

q

$$\frac{(1+\theta_0)^n}{(1+\theta_1)^n} \frac{\prod_{i=1}^{n}(x+\theta_1)^2}{\prod_{i=1}^{n}(x+\theta_0)^2} \le K_\alpha$$

$$\frac{\prod_{i=1}^{n}(x+\theta_1)^2}{\prod_{i=1}^{n}(x+\theta_0)^2} \le K_\alpha \frac{(1+\theta_0)^n}{(1+\theta_1)^n}$$

It cannot be written in the form of sample statistics which is independent of hypothesis. Hence no best critical region exists for such p.d.f.

One thing more is that

“Prayer is free outgoing call to Allah. No network busy and battery problem. Always good signals and all msgs received so plz during your prayer remember us. (Being a teacher)”